

Positronium Formation in  $e^+ + H^-$  Collisions

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## I. ABSTRACT

Cross sections for positronium formation by capture from the negative hydrogen ion are given. Orthogonalization corrections to the Coulomb (First) Born Approximation (CBA) differential and total cross sections are calculated using approximate  $H^-$  wave functions of both Löwdin<sup>1</sup> and Chandrasekhar.<sup>2</sup>

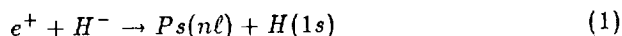
The present calculation of the CBA cross sections using the post interaction for Löwdin's wave function (LCBAPS) disagree with the calculation of Choudhury, Mukherjee, and Sural (CMS), whereas our results using the prior interaction agree. Thus, where CMS found an order of magnitude post-prior discrepancy in the differential cross sections except at forward angles, and a markedly different shape to the minima, the present post and prior results differ by 1% to 10% at 100 eV, and the minima have the same shape and occur within one degree of each other. Chandrasekhar's "open-shell" wave function, which is superior to Löwdin's in bound-state problems since it gives a negative binding energy, gives post and prior cross sections that are almost indistinguishable at this energy and 1/2 to 2/3 as large as the LCBA.

Various methods of orthogonalizing the unbound projectile to the possible bound states are considered. It is found that treating the atomic nuclei as if they were isotopic spin projections<sup>4</sup> of a single type of "nucleon" gives cross sections that are an improvement over the CBA.

## II. INTRODUCTION

Reliable cross sections for the various positronium (Ps) formation processes are essential for an accurate calculation of the width of the .511 MeV annihilation line that has been observed in the region of the galactic center,<sup>5</sup> in solar flares,<sup>6</sup> and in planetary nebulae.<sup>7</sup> In the transition regions of planetary nebulae the concentration of

the negative hydrogen ion<sup>8</sup> should be large enough for the reaction



to make an important contribution to the line width.<sup>9</sup> Furthermore, because this reaction is exothermic, it appears to be the dominant mechanism for positronium formation at energies below the 6.8 eV positron kinetic energy threshold for electron capture from neutral hydrogen even in regions where the  $H^-$  density is low.

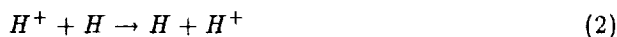
The present calculation relies on the exact treatment of the three species of bound states inherent in Fock-Tani representation. Also included is the further presumption,<sup>4</sup> which produced remarkable agreement between the orthogonalized first order calculation<sup>11</sup> of charge transfer from hydrogen and the (presumably exact) variational result,<sup>12</sup> of treating the proton and positron as isospin-like projections (of different mass) of a single species of "nucleon."

## III. ORTHOGONALIZATION

In scattering processes involving bound states, one must subtract the projection of the translational states of free particles onto the corresponding bound states if the contribution of these particles to the amplitude is not to be counted twice. Fock-Tani representation<sup>13</sup> has been a powerful tool for generating these orthogonalization corrections. In this representation the reactants, intermediate states, and products are treated symmetrically, and composites are treated exactly within a single second-quantized Hamiltonian. Unbound particles are *exactly* orthogonal to bound states, and all interactions contain the proper orthogonalization subtractions so that free particles do not have sufficient energy to bind (this binding energy is accounted for in the asymptotic Hamiltonian), and assuring that there is no double counting.

Because the Lippmann-Schwinger series for the Fock-Tani T-matrix contains higher order contributions at each

order than does the standard Born series, one has the hope of improved results at each order. Ojha *et al.*<sup>14</sup> have calculated the first-order Fock-Tani cross sections for the reaction

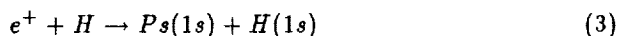


and have obtained good agreement with experiment<sup>15</sup> for differential angles within 1 mrad of the forward direction at 25, 60, and 125 keV and for total cross sections at energies greater than 10 keV. They noted that the orthogonalization correction substantially cancels the internuclear potential. Straton<sup>16</sup> has shown that excluding these p-p terms yields Fock-Tani cross sections that are 18% smaller than when these terms are included. In contrast, the Brinkman-Kramers result,<sup>17</sup> which excludes the p-p term, is 1000% larger than the first Born total cross section that includes this term.<sup>18</sup> Thus the Fock-Tani Hamiltonian produces substantial agreement at first order between experiment and Wick's observation<sup>18</sup> that the internuclear potential should play a negligible role in exact calculations of this process.

Finally, it may be seen that the first-order Fock-Tani differential cross section is virtually identical to that of the second-order boundary-corrected Born approximation<sup>19</sup> (B2B) at 125 keV. This correspondence both affirms the appropriateness of testing the lowest-order Fock-Tani theory in problems in which generic first-order theories would not be expected to be reliable, and requires a deeper study of the question of why it should do so well. In particular, is there a fundamental relation between the orthogonalization process that produces free-particle (continuum) states by subtracting off their Coulomb projections onto the bound states, and cancels the internuclear interaction in the scattering region, and the Coulomb boundary correction process that gives the correct asymptotic states?

#### IV. ISO-ORTHOGONALIZATION

The obvious region in which a first-order theory might not be expected to be reliable is at low energies. Straton<sup>16</sup> has calculated the Fock-Tani total cross section for the reaction



and obtained a result that was larger than the first Born approximation (FBA), whereas the (presumably exact) variational result of Brown and Humberston<sup>12</sup> was smaller than the FBA.

This failure was due to an anomaly of the product form of the Fock-Tani transformation, which does not produce orthogonalizations with respect to all species of bound states. This was immaterial in the reaction (2) since the initial and final bound-state species were identical.

Straton and Girardeau<sup>4</sup> were able to generalize the Fock-Tani transformation on the two-nucleon, one-electron

Hilbert space to produce a T-matrix for either (2) or (3) that was post-prior symmetrical. This was accomplished by thinking of the two atomic nuclei as isospin-like projections of a single species of "nucleon," just as in nuclear physics it is useful to think of the proton and the neutron as isospin projections of a single species of nucleon. The consequence of this way of viewing the system is an up-leveling of the nucleon-exchange contribution to the scattering (elastic and inelastic) amplitude, as in Fig. 1,

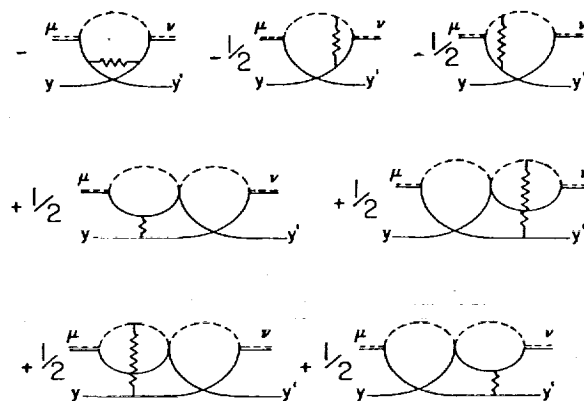


FIG. 1. Proton exchange terms in the Coulomb and orthogonalization interactions in proton-hydrogen scattering. The solid lines are proton propagators, the dashed line is the electron propagator, and the doubled line is the hydrogen atom (composite elementary particle) propagator. Time flows right to left so that the first term represents breakup of a bound-state with quantum numbers  $\nu$  followed by formation of bound-state  $\mu$  due to interaction with the exchanged proton. The last two terms contain the post and prior orthogonalization projectors  $\propto$ .

to a reactive amplitude. Since exchange essentially amounts to a reactive process, it is not surprising that Fig. 1 may be promoted to a reactive matrix element by promoting an "effective" difference between incoming and outgoing free particles to a true difference through the use of an isotopic spin formalism.

Girardeau and Lo<sup>11</sup> applied this iso-orthogonalized matrix element to reaction (3) with superb agreement with the variational result of Brown and Humberston<sup>12</sup>, reproduced in Table I.

Note that Fig. 1 is the average of the post and prior interaction amplitudes, but that neither the post or prior Fock-Tani probabilities, nor the average of the probabilities without interference gives a good result in Table I.

## V. THE FOCK-TANI HAMILTONIAN FOR TWO NUCLEONS AND TWO ELECTRONS

One may develop a Fock-Tani Hamiltonian for a system that contains two nucleons and two electrons using the product form of the transformations that orthogonalize to the three bound species in (1). By working in a coordinate system in which one atomic nucleus is fixed at the origin, and therefore ceases to be a dynamic particle,<sup>16</sup> the unitary operator that transforms the Fock Hamiltonian into the subspace in which the three bound states may be treated as elementary particles may be compounded by the product

$$\hat{U} = \hat{U}_A \hat{U}_B \hat{U}_E, \quad (4)$$

where  $A = Ps$ ,  $B$  is the state with two electrons bound to the origin, and  $E$  is the state with one electron bound to the origin, as in Fig. 2.

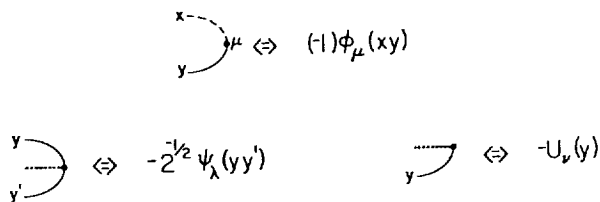


FIG. 2. Diagram correspondences for the positronium wave function, the states in which two electrons are bound to the origin, and the state in which one electron is bound to the origin. The solid lines represent electron propagators, the dashed line represents a positron propagator, and the non-dynamical nucleus fixed at the origin is represented by the dotted line.

Then by interchanging the meanings of the electron and proton propagators in the Hamiltonian given by Straton and Girardeau<sup>4</sup>, the first and most difficult transformation is at hand. Under the second transformation, the electron propagator transforms as in Fig. 3.

$$\hat{U}_B^{-1} \text{ (diagram)} = \text{diagram} - 2 \text{ (diagram)} - 2 \text{ (diagram)}$$

FIG. 3. Transformation orthogonalizing the electron propagator to the states in which two electrons are bound to the origin. The triple line represents the (composite elementary particle) 2-electron bound state propagator.

Under the third transformation, the electron propagator transforms as in Fig. 4



$$\hat{U}_E^{-1} \cdot y \xrightarrow{\hat{U}_E} y \xrightarrow{\cdot} \frac{y}{\cdot} \xrightarrow{\cdot} \frac{y}{\cdot}$$

FIG. 4. Transformation orthogonalizing the electron propagator to the states in which one electron is bound to the origin. The double line represents the (composite elementary particle) 1-electron bound state propagator.

Then the (product form) Fock-Tani Hamiltonian on the 2-nucleon, 2-electron Hilbert space is given in Fig. 5 and Fig. 6.

$$\hat{U}^{-1} \hat{H}_F \hat{U} = \hat{U}_E^{-1} \hat{H}_B \hat{U}_E = H_0 + V_{01} + V_{02} + V_{11} + V_{12}$$

$$H_0 = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + 4 \text{[diagram 6]} + \text{[diagram 7]}$$

$V_{OI} =$   

$$V_{02} = \text{Diagram 1} + \text{Diagram 2} + \text{H.C.}$$

+ 2

$$V_{II} = \text{Diagram 1} + \text{Diagram 2} + \text{H.C.}$$

FIG. 5. The Fock-Tani Hamiltonian on the 2-nucleon, 2-electron Hilbert space. All free propagators are integrated over and all bound state propagators are summed over. The bound state energy is indicated by the  $\sim$ .

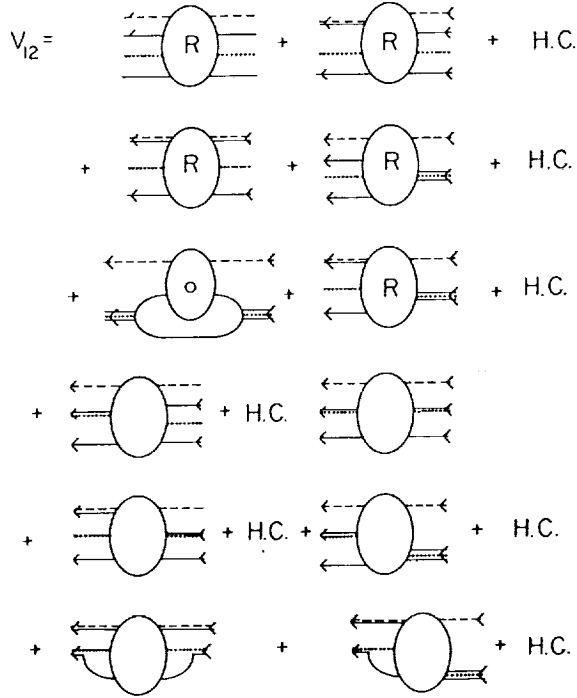


FIG. 6. The 2-nucleon, 2-electron Fock-Tani interaction terms.

The oval in the last term in Fig. 6, which is the interaction potential for (1), is given explicitly in Fig. 7. The second oval in Fig. 7 is given by Fig. 8. The first oval in Fig. 6 is given by the first four terms of Fig. 8 with the  $y'$  propagator replaced by the dotted line representing the nucleon fixed at the origin.

Thus the algebraic translation of Fig. 8 in Fig. 7, after the asymptotic states select the bound state quantum numbers from the sums, is<sup>4,20</sup>

$$\begin{aligned}
 & \int dX dX' dX'' (\delta(X' - X'') [V(XX') + V(X'x)] \phi_\mu(Xx) \\
 & - \delta(X' - X'') \int dY dy \phi_\mu^*(Yy) [V(X'Y) + V(X'y)] \\
 & \times \Delta(Yy, Xx) + \int dy \phi_\mu^*(X'y) [H(X'y) \Delta(X''y, Xx) \\
 & + \frac{1}{2} \Delta(X''y, Xx) H(Xx)] + \int dy \phi_\mu^*(X'y) \left[ \frac{1}{2} V(X'x) \right. \\
 & + V(X'X'') + \frac{1}{2} V(XX') + V(X''y) \left. \right] \Delta(X''y, Xx) \\
 & \times \psi_\nu(XX') U_\lambda(X')
 \end{aligned} \quad (5)$$

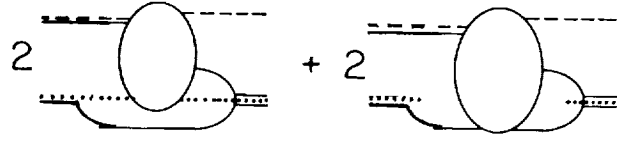


FIG. 7. The last term in 6 in more detail.

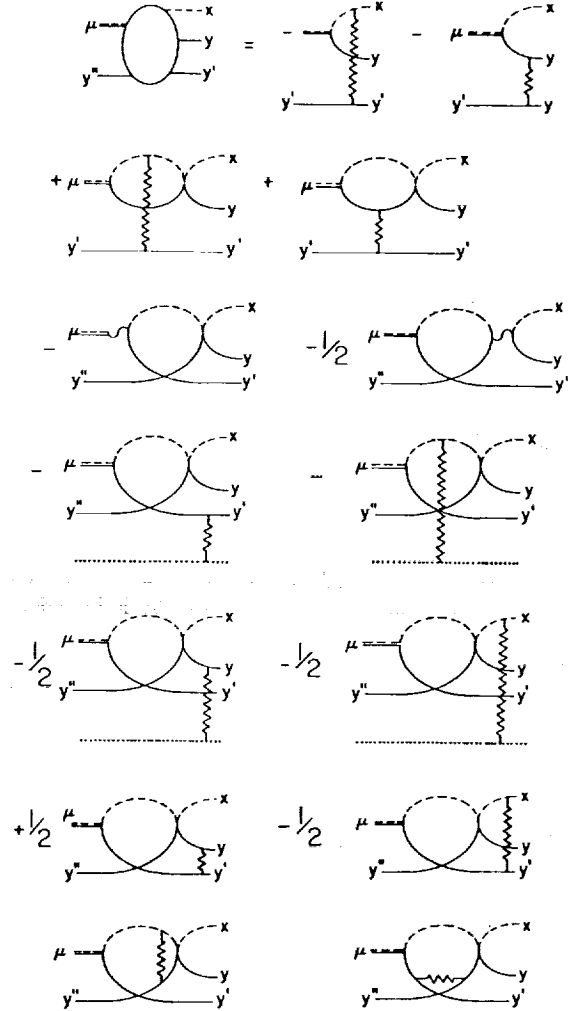


FIG. 8. The second oval in Fig. 7. The zigzag line represents both the sum of Coulomb interactions and the inertial potentials<sup>16</sup> (sometimes called "mass-polarization" terms) experienced by all other particles due to the accelerated reference frame in which one nucleon is constrained to remain at the origin. Crossed fermion lines yield a factor of -1.

The analytic reduction of the Coulomb terms, in which the two electron wave function has been approximated by Löwdin's wave function<sup>1</sup> (with parameters  $\alpha = .4228$ ,  $A = .30025$ ,  $\beta = .9794$ , and  $B = 1.0001$ ), has been outlined in CMS<sup>3</sup>. The derivation for Chandrasekhar's wave

function<sup>2</sup> (with parameters  $a=.28309$ ,  $b=1.03925$ , and normalization  $N=.39513$ ) is identical except that only the cross terms AB are nonzero.

The reduction of the direct-orthogonalization terms, the third and fourth terms in Fig. 8 is much more difficult because of the extra three-dimensional integral. The  $X'$  integral may be done directly giving four terms in the pairings of  $V_\nu V'_{\nu'}$

$$T_{V_\nu V'_{\nu'}} = \sum_\gamma \left[ T_{V_\nu V'_{\nu'}}^{a1\gamma} - (-1)^{\ell_\gamma} T_{V_\nu V'_{\nu'}}^{1a\gamma} \right] \quad (6)$$

where  $a$  is the ratio of the nucleon to electron masses in the final bound state, and the sum is over all possible final bound states. In positronium this ratio is one so that only the odd-parity terms in the sum are nonzero, as was found<sup>16</sup> for the orthogonalization corrections in reaction (3). In the results below, only the 2p contributions are included since the 3p contributions for the similar terms in (3) were negligible. The  $T$ 's are

$$\begin{aligned} T_{V_\nu V'_{\nu'}}^{ab\gamma} = & -\frac{2\sqrt{2}V V'(a+b)^3\lambda^{3/2}}{(2\pi)^3\sqrt{\pi}(\lambda+\nu)^3} \\ & \times \int d\mathbf{x} d\mathbf{r} d\mathbf{s} e^{-i\mathbf{k}_f \cdot [\mathbf{s} + (\eta+\zeta)\mathbf{x}]} u_{1s}^{\mu*}(\mathbf{r}) \\ & \times \exp \left[ -\frac{\lambda+\nu}{\eta(a+b)} | \mathbf{s} + (\eta+\zeta)\mathbf{x} - \eta\mathbf{b}\mathbf{r} | \right] \\ & \times \left[ \frac{\eta(a+b)}{| \mathbf{s} + (\eta+\zeta)\mathbf{x} - \eta\mathbf{b}\mathbf{r} |} + \frac{\nu+\lambda}{2} \right] \\ & \times u_\gamma^\mu(\mathbf{r}) u_\gamma^{\mu*}(\mathbf{s}/\eta) e^{-\nu'|\mathbf{s}/\eta+\mathbf{x}|} \chi_c^+(\mathbf{k}_i, \mathbf{x}) \end{aligned} \quad (7)$$

where  $\zeta = m_p/(m_p + m_e)$ ,  $\eta = 1 - \zeta$ ,  $\mu = \zeta m_e$ ,  $\lambda = m_t m_e/(m_t + m_e)$ , and  $\chi$  is the Coulomb wave function.

Introducing the Fourier (three dimensional integral) representation of the exponential function and the Yukawa potential allows the  $\mathbf{r}$  integral to be evaluated.<sup>21</sup> One may then introduce a (one dimensional integral) Gaussian transform<sup>22</sup> to evaluate the  $\mathbf{s}$  and  $\mathbf{x}$  integrals, leaving a final expression requiring numerical evaluation of a four-dimensional integral. At low energies and small angles the (momentum) radial,  $\theta$ ,  $\phi$ , and (gaussian)  $\rho$  integrals required 32, 16, 24, and 16 Gaussian points, respectively, which used 11 hours of cpu time per data point on a VAX 750.

The exchange-orthogonalization terms in Fig. 8 involve a mixing of coordinates, seen in the last three lines of (5), that further complicates the analytical reduction of these terms. The minimum number of dimensions to be integrated appears to be five for these terms, which would involve a prohibitive amount of time on conventional computers. However, since these terms are exchange corrections to the direct-orthogonalization corrections, they are expected to be small and will be neglected in what follows.

## VI. ISOSPIN SYMMETRY

Because (1) is similar to, and more complicated than, (3), one would expect that the problems associated with a Fock-Tani Hamiltonian derived using a product transformation for the one-electron case would also arise in using a product transformation for the two-electron case. Indeed, the cancellation of the even-parity orthogonalization terms appears in both cases, and if the positron is replaced by a proton the internuclear Coulomb term is cancelled by the corresponding orthogonalization term/ It is hoped that the ideas behind the correction of these problems in (3), which lead to excellent agreement with the variational result, will likewise give a reliable result for (1).

Girardeau and Straton<sup>10</sup> have been able to formally generalize the Fock-Tani transformation to include any number of nucleons, electrons, and bound-state species, but the exacting process of applying Wick's theorem to produce the Hamiltonian on the 2-nucleon, 2-electron Hilbert space has not been completed. Until this process is completed one must use physical ideas to intuit the result.

One might look at the amplitude, Fig. 1, for reaction (3) and postulate that the desired amplitude for (1) should be the average of the amplitudes derived by the post and prior product transformations. Indeed the prior product form corresponding to (4)

$$\hat{U} = \hat{U}_B \hat{U}_A \hat{U}_E \quad (8)$$

is also allowed (though E before A or B is not because its constituents are a proper subset of the constituents of both A and B).<sup>23</sup> The amplitude for this transition is particularly simple because all of the electron-electron interaction energy is included in the bound states and the internuclear potential does not appear (or one might say that the Coulomb term is exactly cancelled by the orthogonalization term for all masses). It is given in Fig. 9.

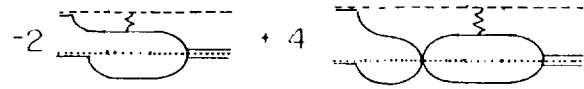


FIG. 9. The prior amplitude for (1).

Evaluation of this amplitude follows that of the CBA closely.

But the fundamental idea that lead to the excellent results for (3) was not post-prior averaging—that was the consequence. The fundamental idea was the treatment of particles of different mass and same charge as if they were isospin projections of a single species of nucleon. Consider Fig. 10.

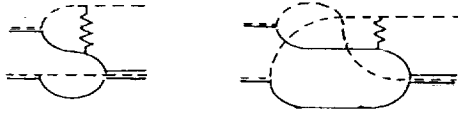


FIG. 10. Direct and (nucleon) exchange Coulomb terms for the electron-nucleon transition amplitude for reaction (1).

If one draws the electron-nucleon interaction diagrams corresponding to the direct and (nucleon) exchange (in a coordinate system in which all four particles are dynamical), it can be seen that the latter may be transformed into the former by a vertical stretching process (multiplying by -1 for each fermion line that is crossed or uncrossed in the process), so that they represent the same physical process. The corresponding direct and exchange orthogonalization projector onto the prior bound states are also equivalent. See Fig. 11.

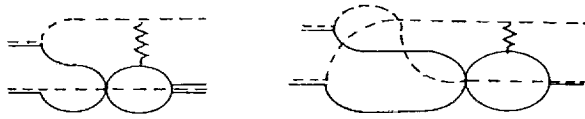


FIG. 11. Direct and (nucleon) exchange prior orthogonalization corrections for the electron-nucleon transition amplitude for reaction (1).

Thus, *isospin symmetry does not imply post-prior symmetry* in reaction (1).

The corresponding direct and exchange orthogonalization projectors onto the upper post bound state are shown in Fig. 12.

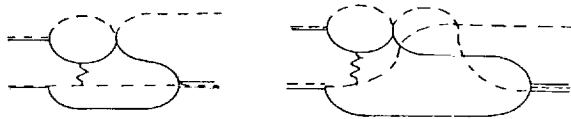


FIG. 12. Direct and (nucleon) exchange corrections orthogonalizing the free nucleon with respect to the upper post bound state, for the electron-nucleon transition amplitude for reaction (1).

These are topologically different and must be treated as two distinct physical processes. Deforming the latter diagram so that the post bound state propagators interchange positions reveals the interpretation of this diagram as the direct orthogonalization projector onto the lower post bound state. Thus the prescription for promoting the exchange amplitude to a reactive amplitude, by promoting the "effective" difference between upper and lower nucleons to a true difference through use of an isotopic spin formalism, leads to an amplitude in which the pro-

jectile is orthogonalized using the average of the direct projectors onto the two post bound states.

Reverting to the coordinate system in which the proton is fixed at the origin, one may show that the direct orthogonalization to the state in which one electron is bound to the origin may be analytically reduced in the same manner as the CBA.

## VII. RESULTS

The differential cross section for reaction (1) is given in Fig. 13 for a positron energy of 100 eV.

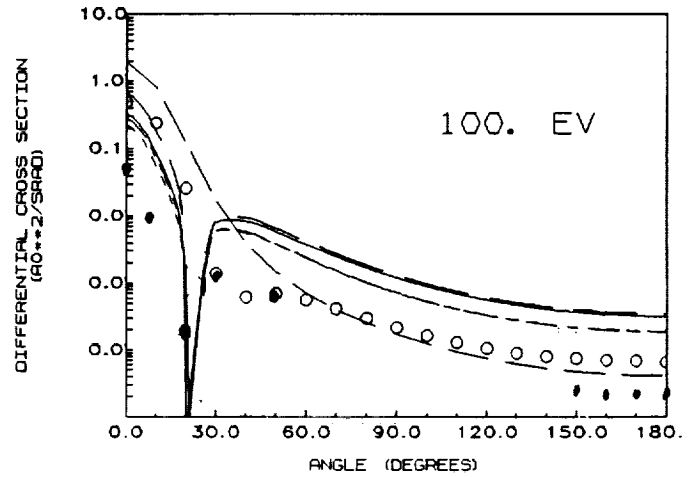


FIG. 13. Electron capture from  $H^-$  into the ground state of positronium. The solid line is the present calculation of the post CBA using Löwdin's wave function (LCBAPS), the open circles are the LCBAPS of CMS<sup>3</sup>, the dashed lines are, in order of decreasing length, the prior LCBA (LCBAPR), the prior direct-orthogonalization result (CDOPR), the CCBAPS using Chandrasekhar's wave function (CCBAPR), and the CCBAPS. The solid points are the iso-orthogonalization correction.

Although the present LCBAPR and the calculation of Choudhury, Mukherjee, and Sural<sup>3</sup> agree, the present calculation of the post CBA cross sections using the post interaction for Löwdin's wave function<sup>1</sup> (LCBAPS) disagree with the calculation of CMS. Where they found an order of magnitude post-prior discrepancy in the differential cross sections except at forward angles, and a markedly different shape to the minima, the present post and prior results differ by 1% to 10% at 100 eV, and the minima have the same shape and occur within one degree of each other.

A cross-check of the present analytic result and their result (which they kindly sent) produced agreement at this stage, so the disagreement is in the computer codes. Four independent reprogrammings, two using an alternate reduction of the integrals giving a different but equivalent analytical result, have reproduced the present results. Ad-

ditionally, there is a "phase space" argument in favor of the present result: that it is less likely that an error would produce nearly identical post and prior curves if they were truly dissimilar than that an error would produce dissimilar curves if they were truly nearly identical.

The CBA results using Chandrasekhar's "open-shell" wave function<sup>2</sup> function gives a binding energy of  $-.522592$  atomic units for  $H^-$ , which is within .4% of the correct value, but Löwdin's wave function does not give a negative binding energy. One would suspect that the former would also yield better results in a scattering problem. It may be seen in Fig. 13 that the post and prior results are almost indistinguishable for the former. Also the magnitude of the CCBA results are  $1/2$  to  $2/3$  as large as the LCBA results, which is expected to exceed the exact result.

The differential cross sections at energies .1, .5, and 1. eV are given in Fig. 14 and the total cross sections are given in Table II. The latter was obtained by a simple extended Simpson's rule from the differential cross sections so the error may be of order 10%, as seen by comparing LCBAPR at 100 eV to the result, .255(-1), of CMS. As noted below, the error due to the approximate  $H^-$  wave function is certainly larger.

Note that the CDIOPS and CDOPS results show some oscillations characteristic of a lack of convergence in the energy region around 90 degrees, but are well converged in at small and large angles, the regions with the greatest contributions to the total cross sections. The LDIOPS is smoother because of the averaging inherent in the larger number of nonzero terms in Löwdin's wave function. It may be possible to redistribute the number of Gaussian integration points among the four integrals to improve the convergence in the central region. But the difference between the LDIOPS and CDIOPS results gives a bound on the accuracy of the approximate wave function that lead to the CDIOPS result and the oscillations are much smaller than this estimate.

It may be seen that all orthogonalization corrections tend to remove the minimum that appears in the CBA results, a minimum that was shown to be spurious in the reaction (2). However, the CDOPR and CDOPS cross sections (and the result obtained by averaging these amplitudes) are larger than both the CCBAPR and CCBAPS cross sections. Since the Coulomb Born approximation for the exact  $H^-$  wave function appears to be larger than the unitarity limit near zero incident energy, one would want cross sections less than the CBA result in this region. The iso-orthogonalization correction gives a result that is less than the CBA in this region.

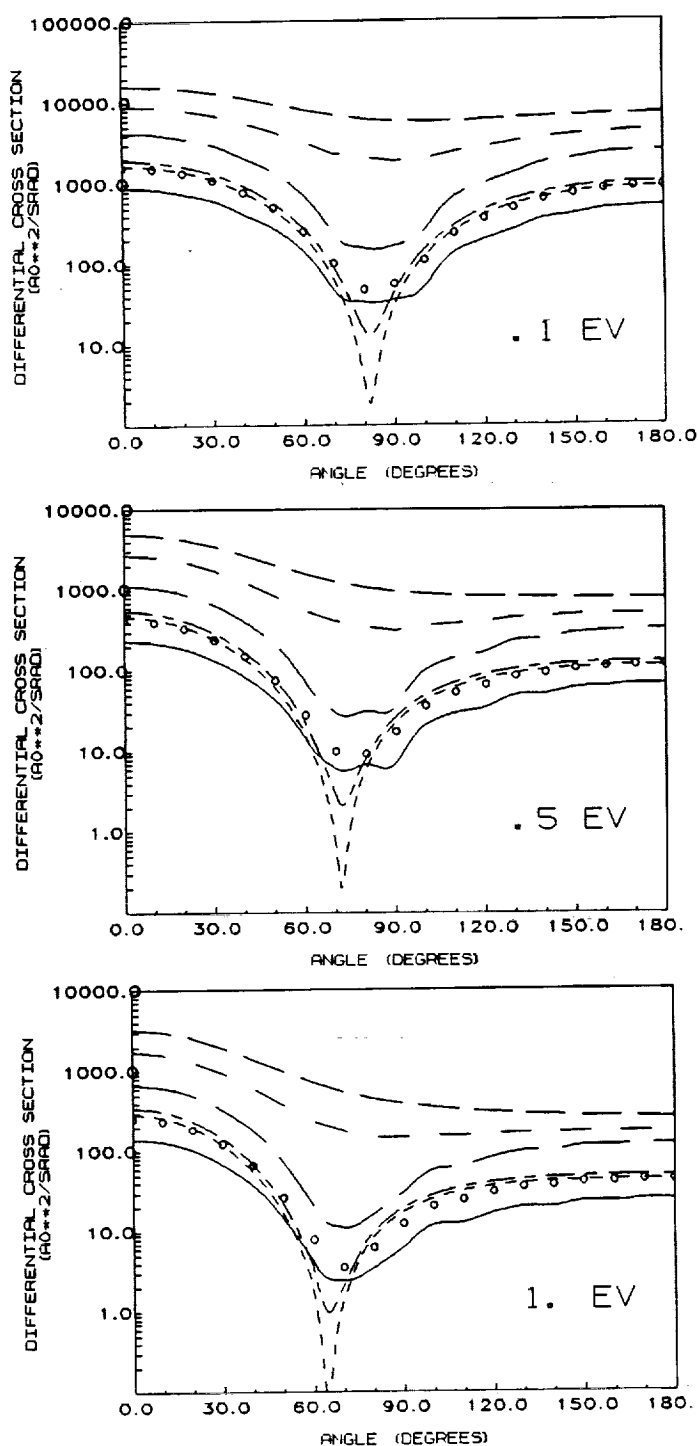


FIG. 14. Electron capture from  $H^-$  into the ground state of positronium using the "open-shell" wave function. The solid line is the present calculation including the (post direct) iso-orthogonalization (CDIOPS) (the open circles are the same result using Löwdin's wave function (LDIOPS)), the dashed lines are, in order of decreasing length, the post direct-orthogonalization result (CDOPS), the prior direct-orthogonalization result (CDOPR), the CCBAPR, and the CCBAPS.

TABLE I. Fock-Tani cross sections for reaction (3) in units of  $\pi a_0^2$ .<sup>11</sup>

Energy (eV)	FBA	Post FT	Prior FT	Average without interference	Symmetric FT	Humberston
6.8	0.032	0.00094	0.011	0.0061	0.0046	0.0032
7.65	1.869	1.454	0.458	0.956	0.74	0.7
8.7	3.344	3.247	0.426	1.836	1.259	1.3
9.2	3.835	3.868	0.457	2.162	1.443	
9.826	4.287	4.447	0.499	2.473	1.653	1.67
10.0	4.385	4.573	0.511	2.542	1.709	
13.6	4.788	5.187	0.979	3.083	2.541	
20.0	3.349	3.631	1.352	2.491	2.278	
30.0	1.651	1.773	1.078	1.425	1.359	
40.0	0.848	0.902	0.681	0.791	0.761	
50.0	0.465	0.489	0.417	0.453	0.436	
60.0	0.269	0.281	0.260	0.271	0.260	
70.0	0.164	0.170	0.166	0.168	0.162	
80.0	0.104	0.107	0.110	0.108	0.104	
90.0	0.068	0.070	0.074	0.072	0.069	
100.0	0.046	0.047	0.052	0.049	0.047	

TABLE II. Total cross sections for electron capture from  $H^-$  into the ground state of positronium, in units of  $\pi a_0^2$ .

E (eV)	LCBAPR	LCBAPS	CCBAPR	CCBAPS	CDOPR	CDOPS	LDIOPS	CDIOPS
.1	.167(4)	.237(4)	.201(4)	.170(4)	.304(5)	.458(4)	.904(3)	.947(3)
.5	.634(3)	.456(3)	.384(3)	.327(3)	.576(4)	.865(3)	.321(3)	.178(3)
1.	.303(3)	.217(3)	.181(3)	.155(3)	.269(4)	.402(3)	.149(3)	.825(2)
100.	.232(-1)	.151(-1)	.112(-1)	.986(-2)	.791(-1)			



## VIII. CONCLUSION

Cross sections for positronium formation by electron capture from the negative hydrogen ion have been calculated in the energy region below the 6.8 eV threshold for capture from hydrogen. The lowest order Born approximation has been augmented by orthogonalization corrections. The present treatment has utilized the perspective of treating the atomic nuclei (of like charge and vastly different mass) as if they were isospin projections of a single species of "nucleon," and has examined the consequences of this perspective. In capture from hydrogen this iso-orthogonalized Fock-Tani result yielded excellent agreement with the variational result. To date there is no variational result for capture from  $H^-$ , due difficult integrals involving Coulomb waves, but the iso-orthogonalized result show promise of yielding a reliable result.

### A. Acknowledgements

Part of this work was done while JCS was a guest scientist conducting research through the National Research Council-NASA Research Associateship Program. He would like to thank Drs. Anand Bhatia and Aaron Temkin for fruitful discussions during this tenure. Part of this work was supported by the Division of Chemical Sciences, Office of Energy Research, U.S. Department of Energy.

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